## Case II :- Homogeneous Type :-

$$\boxed{\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}}$$

If $f_1(x,y)$ and $f_2(x,y)$ have same degree.

Ex- $\frac{dy}{dx} = \frac{\overbrace{x^2}^{\text{degree '2'}} - \overbrace{y^2}^{\text{degree '2'}} + \overbrace{xy}^{\text{degree '2'}}}{\underbrace{y^2}_{\text{deg. '2'}} + \underbrace{xy}_{\text{degree '2'}}} = \frac{\text{Total degree 2}}{\text{Total degree 2}}$

So it is first order Homogeneous differential equation.

**Note :-** "In addition and subtraction degree remains same but in multiplication degree added up and in division degree subtracted."

* Solution of Homogeneous D.E. :-

(1) $\boxed{y = vx} \Rightarrow v = \frac{y}{x} \longleftarrow \sin\frac{y}{x}$

$$\Rightarrow \boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

(2) $x = vy \Rightarrow v = \frac{x}{y} \longleftarrow e^{x/y}$

$$\Rightarrow \boxed{\frac{dx}{dy} = v + y\frac{dv}{dy}}$$

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Q.2 Solve the Differential Equation (Homogeneous type) :-

(iii) $(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$ given that $y(0) = 1$.

So" $(1+e^{x/y})dx + e^{x/y}\left(1+\frac{x}{y}\right)dy = 0$

$$\frac{dx}{dy} = \frac{e^{x/y}\left(\frac{x}{y}-1\right)}{1+e^{x/y}}$$

$$\Rightarrow v + y\frac{dv}{dy} = \frac{e^v(v-1)}{1+e^v}$$

$$\Rightarrow y\frac{dv}{dy} = \frac{e^v(v-1)}{1+e^v} - v$$

$$= \frac{-e^v - v}{1+e^v}$$

$$\Rightarrow \int \frac{e^v+1}{e^v+v}\,dv = -\int \frac{dy}{y}$$

Let $e^v + v = \rho$

$dv(e^v+1) = d\rho$

$$\Rightarrow \ln(e^v+v) = -\ln y + \ln c$$

$$\Rightarrow \ln\left(e^{x/y} + \frac{x}{y}\right) = \ln\left(\frac{c}{y}\right)$$

$$\Rightarrow e^{x/y} + \frac{x}{y} = \frac{c}{y}$$

$\because$ at $x=0$, $y=1$

So $e^0 + 0 = c$

$$\Rightarrow \boxed{c=1}$$

So $\boxed{e^{x/y} + \frac{x}{y} = \frac{1}{y}}$ Ans

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Q.5 Suppose that, a tumour in a rat is approximately spherical and its rate of growth is proportional to its diameter. If the tumour has diameter 5 m.m. when detected at 8mm, after three months later, what will the diameter be after another three months?

(a) 10.1 m.m. (b) 12.8 m.m. (c) 11.6 m.m. (d) 13.2 m.m.